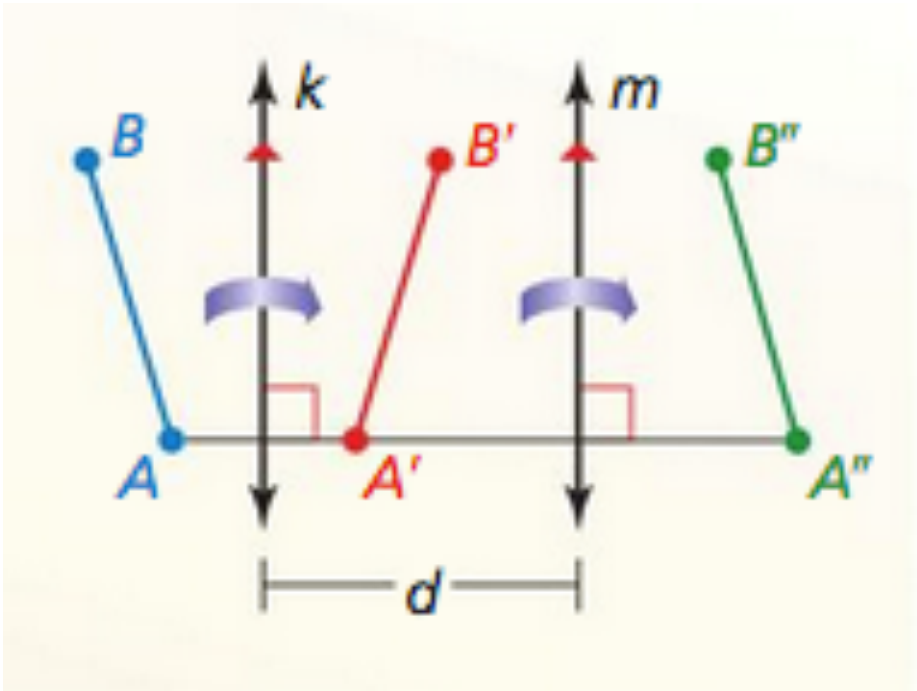
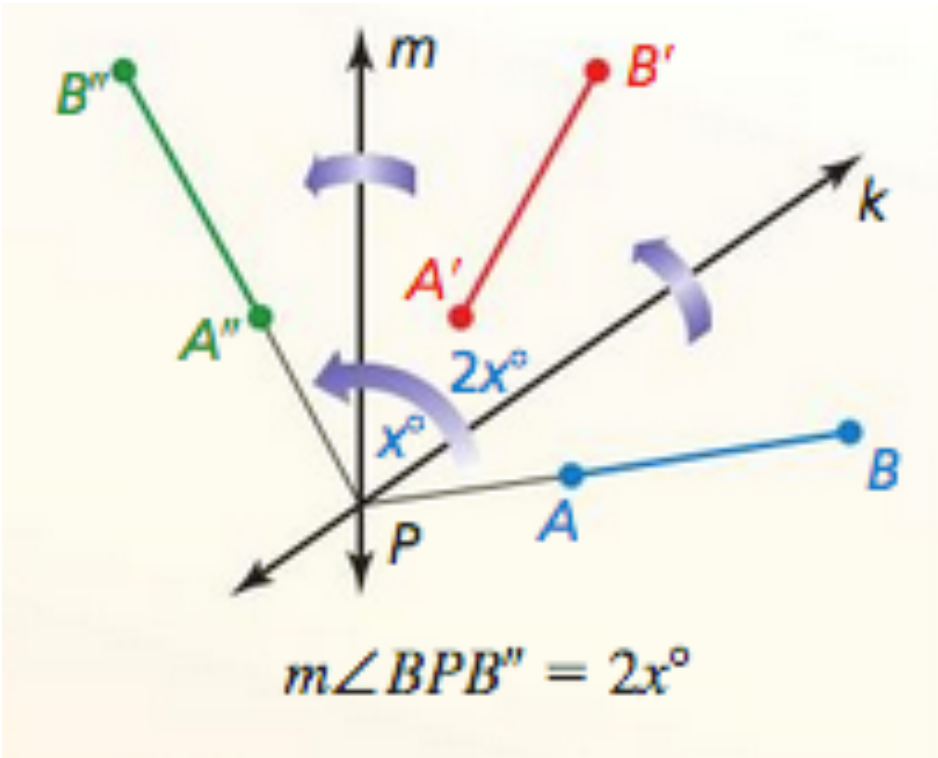

k
m
B
B′
B″
A
A′
A″
d



VOCABULARY

- congruent figures
- congruence transformation

LEARNING TARGETS

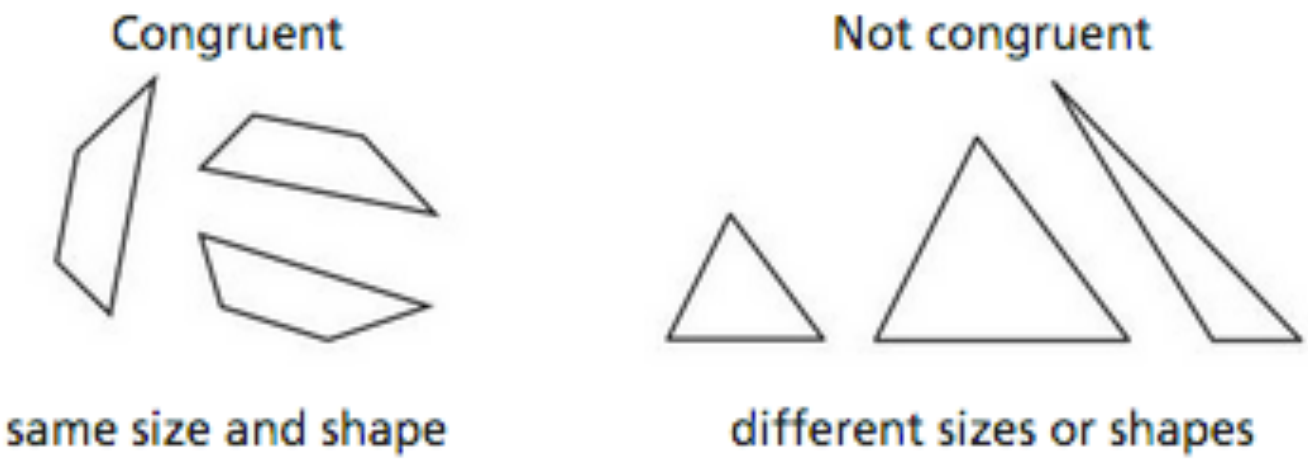
- I can identify congruent figures
- I can describe and perform congruence transformations
- I can use theorems about congruence transformations

4.4 Congruence and Transformations

I can identify congruent figures.

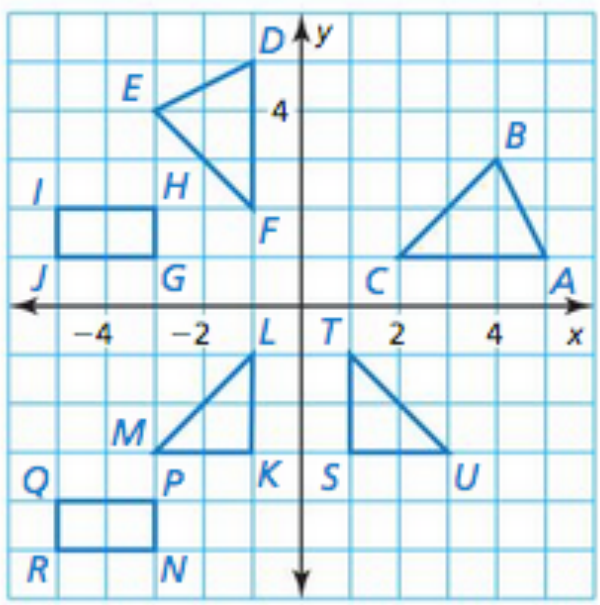
Vocabulary: Congruent Figures

- Two geometric figures are **congruent figures** if and only if there is a rigid motion or a composition of rigid motions that maps one of the figures onto the other. *Congruent figures* have the same size and shape.



Example

- Identify any congruent figures in the coordinate plane. Explain.



For quadrilaterals *IHGJ* and *QPNR* to be congruent, corresponding angles and corresponding sides must be equal. $IH = QP$, $HG = PN$, $GJ = NR$, $JI = RQ$, and all angles are congruent because they are all 90° angles. So, ▭$IHGJ \cong$ ▭$QPNR$. $\triangle LKM$ is a reflection in the y-axis of $\triangle TSU$. So, $\triangle LKM \cong \triangle TSU$. $\triangle DEF$ is a 90° rotation of $\triangle ABC$. So, $\triangle DEF \cong \triangle ABC$.

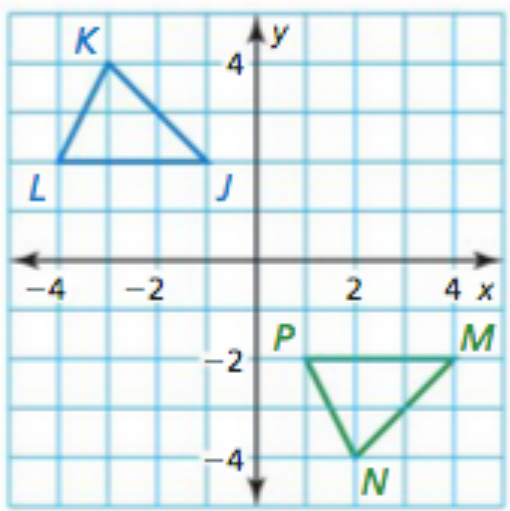
I can describe congruence transformations.

Vocabulary: Congruence Transformation

- Another name for a rigid motion or a combination of rigid motions is a **congruence transformation** because the preimage and image are congruent. The terms “rigid motion” and “congruence transformation” are interchangeable.

Example

Describe a congruence transformation that maps $\triangle JKL$ to $\triangle MNP$.



Sample answer: Using the translation $(x, y) \rightarrow (x + 5, y)$ yields $L(-4, 2) \rightarrow L'(1, 2)$, $J(-1, 2) \rightarrow J'(4, 2)$, and $K(-3, 4) \rightarrow K'(2, 4)$. Reflecting $\triangle L'K'J'$ in the x-axis yields $L'(1, 2) \rightarrow P(1, -2)$, $J'(4, 2) \rightarrow M(4, -2)$, and $K'(2, 4) \rightarrow N(2, -4)$.

I can use theorems about congruence transformations.

Congruence Transformations

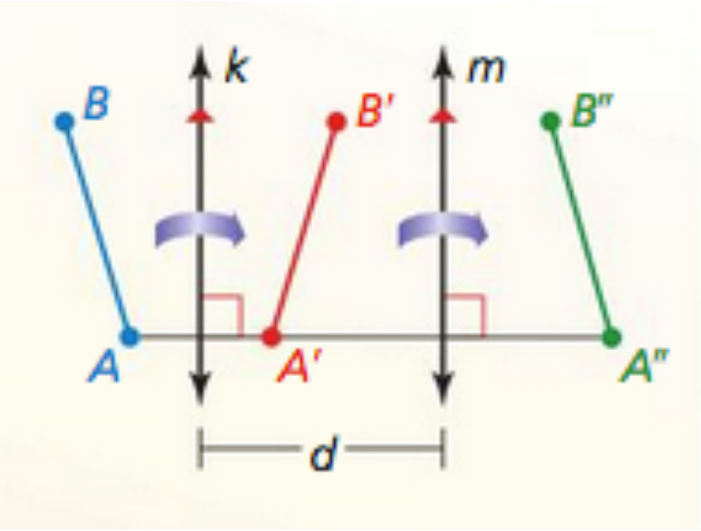
- Compositions of two reflections result in either a translation or a rotation.
    - A composition of two reflections in **parallel lines** results in a **translation**.
    - A composition of two reflections in **intersecting lines** results in a **rotation**.

Theorem 4.2 - Reflections in Parallel Lines Theorem

- If lines *k* and *m* are parallel, then a reflection in line *k* followed by a reflection in line *m* is the same as a translation.

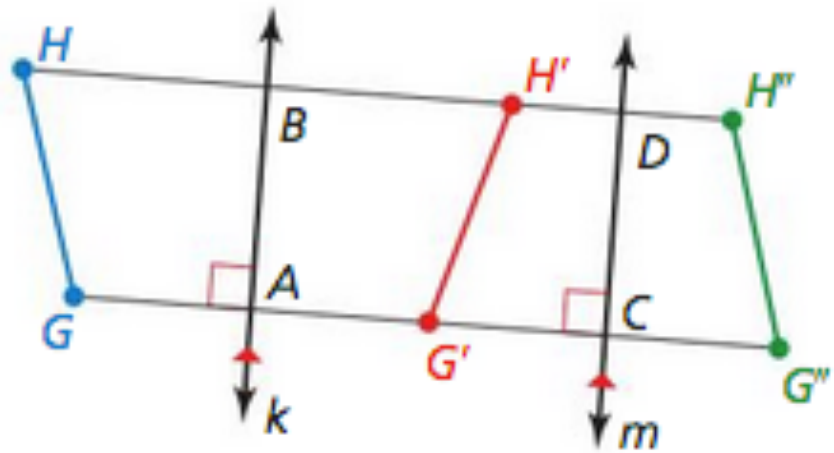
If A'' is the image of A, then

1. $\overline{AA''}$ is perpendicular to k and m, and
2. $AA'' = 2d$, where d is the distance between k and m.



Example

In the diagram, a reflection in line k maps $\overline{GH}$ to $\overline{G'H'}$. A reflection in line m maps $\overline{G'H'}$ to $\overline{G''H''}$. Also, $HB = 9$ and $DH'' = 4$.



a. Name any segments congruent to each segment: $\overline{GH}$, $\overline{HB}$, and $\overline{GA}$.

b. Does $AC = BD$? Explain.

c. What is the length of $\overline{GG''}$?

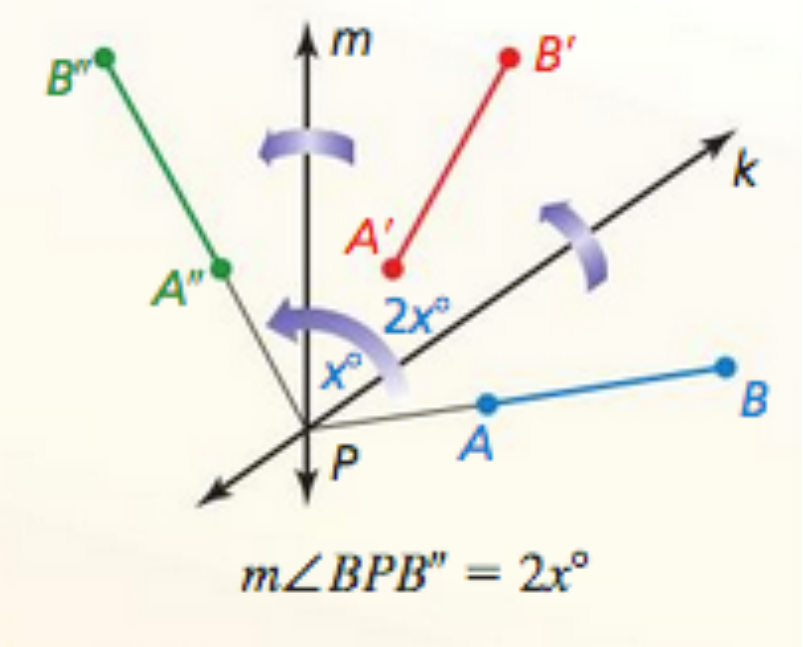
a. $\overline{GH} \cong \overline{G'H'}$, and $\overline{GH} \cong \overline{G''H''}$. $\overline{HB} \cong \overline{H'B}$. $\overline{GA} \cong \overline{G'A}$.

b. Yes, $AC = BD$ because $\overline{GG''}$ and $\overline{HH''}$ are perpendicular to both k and m. So, $\overline{BD}$ and $\overline{AC}$ are opposite sides of a rectangle.

c. By the properties of reflections, $H'B = 9$ and $H'D = 4$. The Reflections in Parallel Lines Theorem implies that $GG'' = HH'' = 2 \cdot BD$, so the length of $\overline{GG''}$ is $2(9 + 4) = 26$ units.

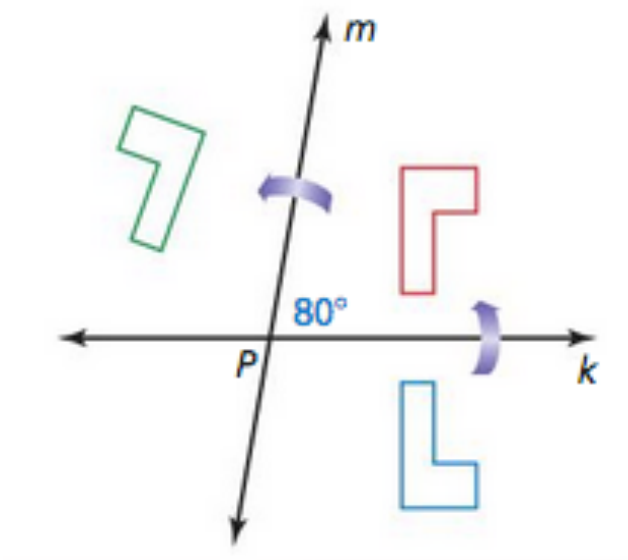
Theorem 4.3 - Reflections in Intersecting Lines Theorem

- If lines *k* and *m* intersect at point P, then a reflection in line *k* followed by a reflection in line *m* is the same as a rotation about point P.
  - The angle of rotation is 2x°, where x° is the measure of the acute or right angle formed by lines *k* and *m*.

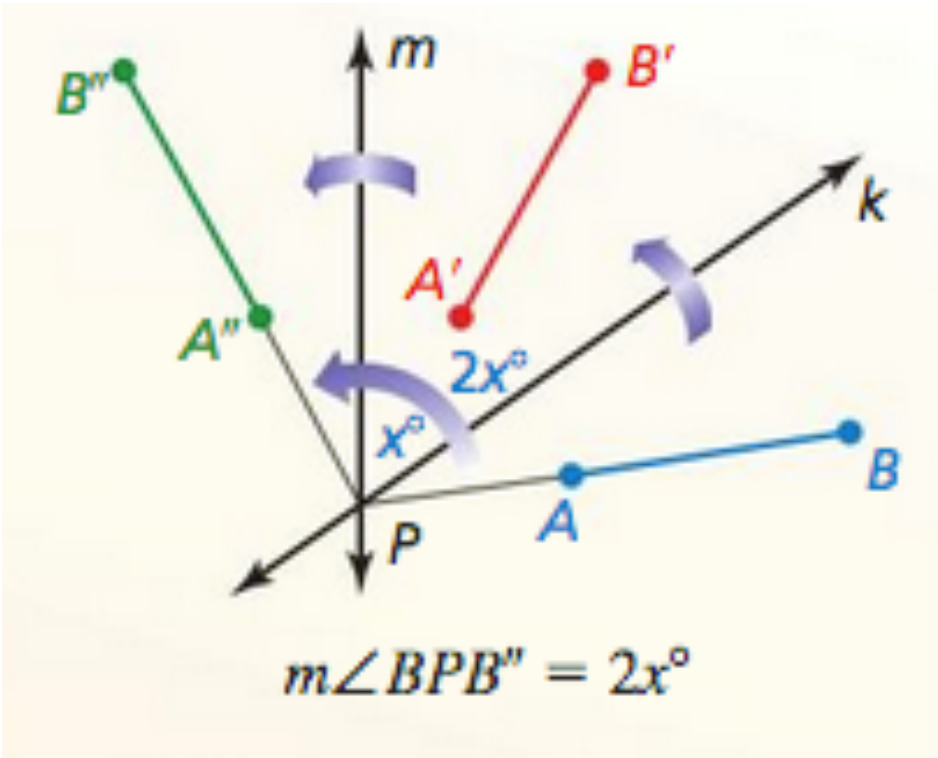


Example

- In the diagram, the premise is reflected in line k, then in line m. Describe a single transformation that maps the blue figure onto the green figure.



Rotate the blue figure 160° (2 • 80) about *P* in order to position the green figure.



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