Conditional		
p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Conditional		
p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

VOCABULARY

- conditional statement
- if-then form
- hypothesis
- conclusion
- negation
- converse
- inverse
- contrapositive
- equivalent statement
- perpendicular lines
- biconditional statement
- truth value
- truth table

LEARNING TARGETS

- I can write conditional statements
- I can work with converse, inverse, and contrapositive statements
- I can write biconditional statements
- I can make truth tables

2.1 Conditional Statements

Vocabulary: Conditional Statement, Hypothesis, and Conclusion

- A **conditional statement** is a logical statement that has two parts, a *hypothesis* p and a *conclusion* q. When a conditional statement is written in **if-then form**, the "if" part contains the **hypothesis** and the "then" part contains the **conclusion**.

If p, then q.

$$p \rightarrow q$$

p implies q

Vocabulary: Negation

- The **negation** of a statement is the opposite of the original statement. To write the negation of a statement p, you write the symbol for negation (~) before the letter.

"not *p*" is written ~*p*

Vocabulary: Converse, Inverse, and Contrapostive

- To write the **converse** of a conditional statement, <u>exchange</u> the hypothesis and conclusion.

  If q, then p.

  $q \rightarrow p$

- To write the **inverse** of a conditional statement, <u>negate</u> both the hypothesis and conclusion.

  If not p, then not q.

  $\sim p \rightarrow \sim q$

- To write the **contrapositive** of a conditional statement, first <u>write the converse</u>. Then <u>negate</u> both the hypothesis and the conclusion.

  If not q, then not p.

  $\sim q \rightarrow \sim p$

Vocabulary: Equivalent Statements

- A conditional statement and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional statement are either both true or both false. In general, when two statements are both true or both false, they are called **equivalent statements**.

Using Definitions
Vocabulary: Perpendicular Lines

- You can write a definition as a conditional statement in if-then form or as its converse. Both the conditional statement and its converse are true for definitions. For example, consider the definition of perpendicular lines.

- If two lines intersect to form a right angle, then they are **perpendicular lines**.

- You can also write the definition using the converse: If two lines are **perpendicular lines**, then they intersect to form a right angle.

Vocabulary: Biconditional Statement

- When a conditional statement and its converse are both true, you can write them as a single *biconditional statement*. A **biconditional statement** is a statement that contains the phrase "if and only if."

p if and only if q

$$p \leftrightarrow q$$

Any definition can be written as a biconditional statement.

Vocabulary: Truth Value and Truth Table

- The **truth value** of a statement is either true (T) or false (F). You can determine the conditions under which a conditional statement is true by using a **truth table**. The truth table below shows the truth value for hypothesis *p* and conclusion *q*.

Conditional		
p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

More on Truth Tables

- The conditional statement $p \rightarrow q$ is only false when a true hypothesis produces a false conclusion.

More on Truth Tables

Consider the statement "If you clean your room, then I promise to give you some ice cream." What could happen here???

- If you clean your room and I give you ice cream, then there's no problem: I kept my promise. The truth value of $p \rightarrow q$ in this case is true.

- If you clean your room and I don't give you ice cream, then I broke my promise and you have a legitimate complaint. The truth value of $p \rightarrow q$ in this case is false.

- If you didn't clean your room and still I gave you ice cream, then I'm just a nice person. The truth value of $p \rightarrow q$ would be true.

- If you didn't clean your room and I didn't give you ice cream … well, that's what you get for not doing what I asked you to do! The truth value of $p \rightarrow q$ is true once again.

***Note that the only time the truth value of $p \rightarrow q$ is false is if I break my promise: the statement p is true (the condition is met) and the statement q is false (no follow-through).

More on Truth Tables

- Two statements are *logically equivalent* when they have the same truth table.

More on Truth Tables

- The converse and inverse are logically equivalent because they have the same truth table.

Converse		
p	q	$q \to p$
T	T	T
T	F	T
F	T	F
F	F	T

Inverse				
p	q	$\sim p$	$\sim q$	$\sim p \to \sim q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

- A conditional statement and its contrapositive are logically equivalent because they have the same truth table.

Conditional		
p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Contrapositive				
p	q	$\sim q$	$\sim p$	$\sim q \to \sim p$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

Conditional		
p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

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2.1 Conditional Statements