

Algebra 1

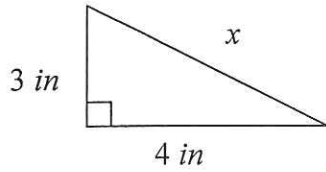
Lesson 11.5

The Distance and Midpoint Formulas

Warm-Up

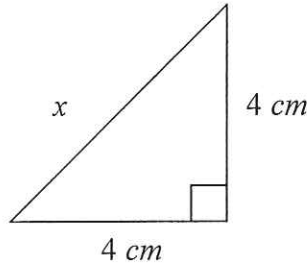
Find the length of the missing side using the Pythagorean Theorem. Be sure all answers are in simplest form.

(a)



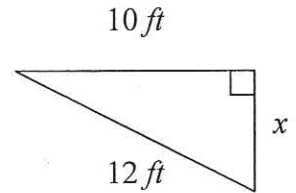
$$\begin{aligned} (3)^2 + (4)^2 &= x^2 \\ 9 + 16 &= x^2 \\ x^2 &= 25 \\ x &= 5 \text{ in} \end{aligned}$$

(b)



$$\begin{aligned} (4)^2 + (4)^2 &= x^2 \\ 16 + 16 &= x^2 \\ x^2 &= 32 \\ x &= \sqrt{32} = \sqrt{16 \cdot 2} \\ x &= 4\sqrt{2} \text{ in} \end{aligned}$$

(c)



$$\begin{aligned} (10)^2 + (x)^2 &= (12)^2 \\ 100 + x^2 &= 144 \\ x^2 &= 44 \\ x &= \sqrt{44} \\ x &= 2\sqrt{11} \text{ in.} \end{aligned}$$

The Distance Formula

The distance d between any two points (x_1, y_1) and (x_2, y_2) is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

-or-

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

Example 1. Find the Distance Between Two Points

Find the distance between the two points. Be sure all answers are in simplest form.

(a) $(-3, 1)$ and $(2, 3)$

$$\begin{aligned} d &= \sqrt{(3 - (-3))^2 + (2 - 1)^2} \\ d &= \sqrt{(2^2 + 5^2)} \\ d &= \sqrt{29} \\ d &= \sqrt{29} \text{ units} \end{aligned}$$

(b) $(-2, 1)$ and $(2, 5)$

$$\begin{aligned} d &= \sqrt{(5 - 1)^2 + (2 - (-2))^2} \\ d &= \sqrt{4^2 + 4^2} \\ d &= \sqrt{32} \\ d &= 4\sqrt{2} \text{ units} \end{aligned}$$

(c) $(2, -4)$ and $(14, 1)$

$$\begin{aligned} d &= \sqrt{(1 - 2)^2 + (1 - (-4))^2} \\ d &= \sqrt{5^2 + 12^2} \\ d &= \sqrt{169} \\ d &= 13 \text{ units} \end{aligned}$$

The Midpoint Formula

The midpoint M of the line segment with endpoints (x_1, y_1) and (x_2, y_2) is:

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Example 2. Finding the Midpoint

Find the midpoint of the line segment with the given endpoints.

(a) $(2, 5)$ and $(6, 1)$

(b) $(-3, -1)$ and $(7, -5)$

(c) $(3, 7)$ and $(14, 20)$

$$M\left(\frac{2+6}{2}, \frac{5+1}{2}\right)$$

$$M\left(\frac{-3+7}{2}, \frac{-1+(-5)}{2}\right)$$

$$M\left(\frac{3+14}{2}, \frac{7+20}{2}\right)$$

$$M(4, 3)$$

$$M(2, -3)$$

$$M\left(\frac{17}{2}, \frac{27}{2}\right)$$

$$M(8.5, 13.5)$$

Try It!

For each set of ordered pairs find the **distance** between the points and find the **midpoint**.

(a) $(4, 2)$ and $(6, 5)$

(b) $(5, 9)$ and $(-1, 6)$

(c) $(6, -1)$ and $(-4, -5)$

$$d = \sqrt{(5-2)^2 + (6-4)^2}$$

$$d = \sqrt{(6-9)^2 + (-1-5)^2}$$

$$d = \sqrt{(-4-6)^2 + (-5+1)^2}$$

$$d = \sqrt{(3)^2 + (2)^2}$$

$$d = \sqrt{(-3)^2 + (-6)^2}$$

$$d = \sqrt{100 + 16}$$

$$d = \sqrt{13} \text{ units}$$

$$d = \sqrt{9 + 36}$$

$$d = \sqrt{116}$$

$$M\left(\frac{4+6}{2}, \frac{2+5}{2}\right)$$

$$d = \sqrt{45} = 3\sqrt{5} \text{ units}$$

$$d = 2\sqrt{29} \text{ units}$$

$$M\left(\frac{10}{2}, \frac{7}{2}\right)$$

$$M\left(\frac{5+(-1)}{2}, \frac{9+6}{2}\right)$$

$$M\left(\frac{6+(-4)}{2}, \frac{-1+(-5)}{2}\right)$$

$$M(5, 3.5)$$

$$M\left(\frac{4}{2}, \frac{15}{2}\right)$$

$$M\left(\frac{2}{2}, \frac{-6}{2}\right)$$

$$M(2, 7.5)$$

$$M(1, -3)$$

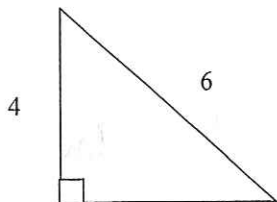
Homework:

Assignment: Pg. 747 #3-10, 22-30

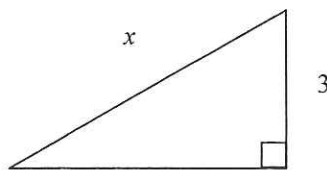
Review:

Find the missing value.

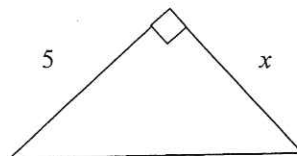
1.



2.



3.



x

8

8