

Algebra 1

Lesson 10.5B

Solve Quadratic Equations by Completing the Square

Warm-Up

Solve by completing the square.

(a) $x^2 + 12x = 28$

$$x^2 + 12x + \boxed{36} = 28 + \boxed{36}$$

$$(x + 6)^2 = 64$$

$$x + 6 = \pm 8$$

$$x = -6 \pm 8$$

$$x = 2, -14$$

(b) $t^2 + 6t + 5 = 0$

$$t^2 + 6t = -5$$

$$t^2 + 6t + \boxed{9} = -5 + \boxed{9}$$

$$(t + 3)^2 = 4$$

$$t + 3 = \pm \sqrt{4}$$

EXACT $\rightarrow t = -3 \pm \sqrt{4}$

APPROXIMATE $t \approx 0.74, -6.74$

(c) $v^2 + 8v = 3$

$$v^2 + 8v + \boxed{16} = 3 + \boxed{16}$$

$$(v + 4)^2 = 19$$

$$v + 4 = \pm \sqrt{19}$$

$$v = -4 \pm \sqrt{19}$$

$$v \approx 0.36, 8.36$$

Completing the Square

$$2x^2 + 20x - 8 = 0$$

$$2x^2 + 20x = 8$$

$$x^2 + 10x = 4$$

$$x^2 + 10x + 25 = 4 + 25$$

$$(x + 5)^2 = 29$$

$$x + 5 = \pm \sqrt{29}$$

$$x = -5 \pm \sqrt{29}$$

1. Write the equation in **quadratic form**: $ax^2 + bx + c = 0$.

2. Move the constant term c to the other side.

3. Divide both sides by the leading coefficient a .

4. **Complete the square.**

5. **Factor** the left hand side. **Simplify** the right hand side.

6. Take the **square root** of both sides.

7. **Solve** for x .

8. **Simplify** the result. (if possible)

To **complete the square**, take the coefficient in front of x , divide it by 2, square it and add the result to **both** sides.

Example 1. Solve by Completing the Square

Solve $3g^2 - 24g + 27 = 0$ by completing the square.

$$3g^2 - 24g = -27$$

$$\frac{3g^2 - 24g}{3} = \frac{-27}{3}$$

$$g^2 - 8g = -9$$

$$g^2 - 8g + \boxed{16} = -9 + \boxed{16}$$

$$(g - 4)^2 = 7$$

$$g - 4 = \pm \sqrt{7}$$

$$g = 4 \pm \sqrt{7}$$

$g = 4 \pm \sqrt{7}$ EXACT

$g \approx 6.65, 1.35$ APPROXIMATE

Example 2. Solve by Completing the Square

Solve $3x^2 - 18x + 9 = 0$ by completing the square.

$$\frac{3x^2 - 18x}{3} = \frac{-9}{3}$$

$$x^2 - 6x = -3$$

$$x^2 - 6x + \boxed{9} = -3 + \boxed{9}$$

$$(x-3)^2 = 6$$

$$x-3 = \pm\sqrt{6}$$

$x = 3 \pm \sqrt{6}$ EXACT

$x \approx 5.45, 0.55$ APPROXIMATE

Example 3. Solve by Completing the Square

Solve $w^2 - 5w = \frac{11}{4}$ by completing the square.

$$w^2 - 5w + \boxed{\frac{25}{4}} = \frac{11}{4} + \boxed{\frac{25}{4}}$$

$$(w - \frac{5}{2})^2 = \frac{36}{4}$$

$$(w - \frac{5}{2})^2 = 9$$

$$w - \frac{5}{2} = \pm 3$$

$$\quad + \frac{5}{2} \quad + \frac{5}{2}$$

$$w = \frac{5}{2} \pm 3$$

$$w = 5.5, -0.5$$

$w = 5.5, -0.5$

Example 4. Solve by Completing the Square

Solve $s^2 - 15s + 8 = -s^2 + 13s$ by completing the square.

$$s^2 - 13s + 8 = 0$$

$$2s^2 - 26s + 8 = 0$$

$$\frac{2s^2 - 26s}{2} = \frac{-8}{2}$$

$$s^2 - 13s = -4$$

$$s^2 - 13s + \boxed{49} = -4 + \boxed{49}$$

$$(s-7)^2 = 45$$

$$s-7 = \pm\sqrt{45}$$

$s = 7 \pm \sqrt{45}$ EXACT

$s \approx 13.71, 0.29$ APPROXIMATE

We'll learn how to simplify this more in Chapter 11

Assignment

New: Pg. 666 #26, 27, 29, 30, 32, 36

Review:

Solve by the method of your choice (factoring or square roots).

1. $x^2 - 7 = 18$

2. $2x^2 + 12 = 30$

3. $3x^2 + 25 = 16$