Algebra 1

Lesson 10.5A

Solve Quadratic Equations by Completing the Square

Warm-Up

Factor each of the following.

(a) $x^2 - 12x + 36$

$(x-6)(x-6)$

$(x-6)^2$

(b) $x^2 + 6x + 9$

$(x+3)(x+3)$

$(x+3)^2$

(c) $x^2 - 16x + 64$

$(x-8)(x-8)$

$(x-8)^2$

Completing the Square

$2x^2 - 12x - 14 = 0$

$2x^2 - 12x = 14$

$x^2 - 6x = 7$

$x^2 - 6x + 9 = 7 + 9$

$(x-3)^2 = 16$

$x - 3 = \pm 4$

$x = 3 \pm 4$

$x = 7, -1$

1. *Write the quadratic equation in **standard form**: $ax^2 + bx + c = 0$.*
2. *Move the constant term **c** to the other side.*
3. *Divide both sides by the leading coefficient **a** if it is something other than 1.*
4. ***COMPLETE THE SQUARE.***
5. ***Factor** the left hand side. **Simplify** the right hand side.*
6. *Take the **square root** of both sides.*
7. ***Solve** for x.*
8. ***Simplify** the result.*

*To **complete the square**, take the coefficient in front of x, divide it by 2, square it and add the result to **both** sides.*

Example 1. Solving by Completing the Square

Solve $x^2 + 6x = 7$ by completing the square.

$x^2 + 6x + \boxed{9} = 7 + \boxed{9}$

$\sqrt{(x+3)^2} = \sqrt{16}$

$x + 3 = \pm 4$

$-3 \quad -3$

$x = -3 \pm 4$

$-3+4 \qquad -3-4$

$x = 1, -7$

Check your answers by solving $x^2 + 6x = 7$ by factoring.

$-7 \quad -7$

$x^2 + 6x - 7 = 0$

$(x+7)(x-1) = 0$

$x = -7, 1$

Example 2. Solving by Completing the Square

Solve $x^2 - 2x - 3 = 0$ by completing the square.

Check your answer by solving $x^2 - 2x - 3 = 0$ by factoring.

$+3 \quad +3$

$x^2 - 2x = 3$

$x^2 - 2x + \boxed{1} = 3 + \boxed{1}$

$\sqrt{(x-1)^2} = \sqrt{4}$

$x - 1 = \pm 2$

$+1 \quad +1$

$x = 1 \pm 2$

$x = 3, -1$

$(x-3)(x+1) = 0$

$x = 3, -1$

Example 3. Solving by Completing the Square

Solve $m^2 + 10m = -8$ by completing the square.

Check your answer by solving $m^2 + 10m = -8$ by factoring.

$m^2 + 10m + \boxed{25} = -8 + \boxed{25}$

$\sqrt{(m+5)^2} = \sqrt{17}$

$m + 5 = \pm\sqrt{17}$

$-5 \quad -5$

$m = -5 \pm \sqrt{17}$ EXACT

APPROXIMATE

$m \approx -0.88, -9.12$

$+8 \quad +8$

$m^2 + 10m + 8 = 0$

NOT FACTORABLE

Example 4. Solving by Completing the Square

Solve $2b^2 - 16b - 24 = 0$ by completing the square.

Check your answer by solving $2b^2 - 16b - 24 = 0$ by factoring.

$+24 \quad +24$

$\frac{2b^2 - 16b}{2} = \frac{24}{2}$

$b^2 - 8b = 12$

$b^2 - 8b + \boxed{16} = 12 + \boxed{16}$

$\sqrt{(b-4)^2} = \sqrt{28}$

$b - 4 = \pm\sqrt{28}$

$+4 \quad +4$

$b = 4 \pm \sqrt{28}$ EXACT

$b \approx 9.29, -1.29$

APPROXIMATE

$2(b^2 - 8b - 12) = 0$

can't FACTOR any MORE

Assignment

New: Pg. 666 #12 – 17, 21, 28

Review:

Simplify.

1. $\frac{9x^2y^2}{3x^2y^5}$

2. $\frac{3c^4}{2d^2} \bullet \frac{d^5}{6c^2}$

3. $\frac{(-2u^3v^2)^2}{12u^2v^4}$